Introduction to Radio Astronomy (WBAS14001)

Final examination – Wednesday 1 February 2017 (09:00 – 12:00)

This examination contains 5 questions worth 20 points each. All questions should be answered. For full credit, all working and calculations should be shown, and full explanations given where required. Numerical answers should be in S.I. units, unless otherwise stated. Below is a list of constants and identities that may be used.

1. (a) The atmospheric window that can be observed from the ground has a low-frequency cut-off. Describe what interactions in the atmosphere result in this cut-off and describe two effects (and their approximate frequency range) that the atmosphere has on radio waves at frequencies less than 1 GHz [6 points].

(b) The atmosphere also affects the transmission of radio waves at higher frequencies. Describe four factors that contribute to the varying transparency of the atmosphere at short radio wavelengths (1 to 100 GHz) and state their dependence on frequency [4 points].

(c) Draw the approximate *total atmospheric transparency* between 1 to 100 GHz, clearly labelling the two ranges in frequency where the atmospheric transparency to radio waves decreases most strongly [2 points].

(d) An observation at a frequency of 1 GHz is made of the supernovae remnant Cassiopeia A with a square horn antenna of diameter 5-m, that has a constant illumination pattern. Given that Cassiopeia A is at a distance of 3.4 kpc and has a diameter of 3 pc, show whether the target is resolved or unresolved by this horn antenna [4 points].

(e) The spectral brightness of an object in the low-frequency regime is defined as,

$$B_\nu(T) = \frac{2kT}{\lambda^2} \tag{1}$$

where the symbols have their usual meaning.

The observation described in part (d) is carried out using a *Dickie-switching radiometer* for an atmosphere with an optical depth of $\tau = 0.01$, and the brightness temperature of the source is found to be 35500 K. Assuming that the source has a constant circular surface brightness distribution of diameter defined in part (d), calculate the intrinsic spectral brightness, flux-density and the luminosity density of Cassiopeia A at 1 GHz [4 points].

2. (a) Define (in words) what is meant by the *radiation resistance* of a dipole antenna [2 points].

(b) Given that the time-averaged radiated power of a short dipole antenna with a varying current distribution is,

$$\langle P \rangle = \frac{\pi}{12\, c\, \epsilon_0} \left(\frac{I \Delta l}{\lambda} \right)^2 \tag{2}$$

where the symbols have their usual meaning, show that the radiation resistance of such an antenna is given by,

$$\langle R_{\rm rad} \rangle = \frac{\pi}{6\, c\, \epsilon_0} \left(\frac{\Delta l}{\lambda} \right)^2 \quad \text{[4 points].} \tag{3}$$

(c) Calculate the radiation resistance for a short dipole antenna of length 30 cm for an observation carried out at a wavelength of 60 cm [2 points].

(d) Hence, describe why short antennas are typically inefficient emitters of electromagnetic radiation [2 points].

(e) Show, for a dipole with a matched load and no-Ohmic losses, that half of the absorbed power is immediately re-radiated, such that,

$$P_{\text{rad}} = \frac{1}{2} P_{\text{in}} \tag{4}$$

where the symbols have their usual meaning [10 points].

3. (a) Describe what is meant by the power pattern of an aperture [2 points].

(b) Given that, for a one-dimensional filled aperture of length x, the electric field pattern in the far-field is the Fourier transform of the electric field illuminating (current grading) the aperture, such that,

$$f(l) = \int_{-\infty}^{+\infty} g(u)\, e^{-i2\pi l u}\, du, \tag{5}$$

where $u = x/\lambda$ and $l = \sin\theta$, show that for a one-dimensional aperture of diameter $D \gg \lambda$ and with a constant current grading, the normalised power pattern is given by,

$$P_n = \text{sinc}^2\left(\frac{\theta D}{\lambda}\right) \quad \text{[10 points].} \tag{6}$$

(c) Draw a schematic diagram of the resulting power pattern of this one-dimensional aperture as a function of angle l, clearly showing the angle that corresponds to the full width at half maximum and the first null [4 points].

(d) Describe a method that can limit the side-lobe structure of this power pattern and state two disadvantages of this method and one advantage (in addition to limiting the side-lobe structrue) [4 points].

4. (a) Draw a schematic diagram of a two-element multiplying interferometer, and describe what each part does to the input radio signal [6 points].

(b) Define (in words) the *geometric delay* of a two-element interferometer and derive how it is related to the separation of the two-elements [4 points].

(c) Define (in words) what is meant by the *primary beam*, *synthesised beam* and *delay beam* of a two -element interferometer, and draw a schematic diagram showing the relative power pattern of each [4 points].

(d) The ALMA interferometric array has 50 antennas that have a diameter of 12-m and an aperture efficiency of $\eta = 0.8$. What is the effective area of a single ALMA antenna [2 points].

(e) The point-source sensitivity of an interferometer is given by,

$$\sigma_{\text{rms}} = \frac{2\,k\,T_{\text{sys}}}{A_{\text{eff}}\sqrt{N(N-1)\Delta\nu\, t_{\text{int}}}}, \tag{7}$$

where the symbols have their usual meaning. Given that the ALMA interferometric array has 50 antennas, with an effective area as found in part (d), a bandwidth of 2 GHz and a system temperature of 100 K, how long would an observation need to be for a 5σ detection of a 100 μJy point source [4 points].

5. (a) Draw a schematic diagram for the spectral energy distribution of a typical star-forming galaxy between 1 GHz and 1000 GHz, showing clearly the main emission mechanisms and their dependance on frequency [6 points].

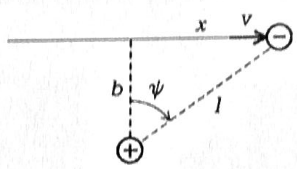
(b) Describe how each of these mechanisms results in the emission of electro-magnetic radiation at radio wavelengths [3 points].

(c) The radio galaxy Cygnus A has a flux density of $S_{325\ \mathrm{MHz}} = 6000$ Jy and $S_{1400\ \mathrm{MHz}} = 1600$ Jy at 325 MHz and 1.4 GHz, respectively. Calculate the spectral index of Cygnus A between these two frequencies and determine whether this is consistent with a thermal or non-thermal emission process [2 points].

(d) The power emitted by an accelerated charge is given by Larmor's formula,

$$P = \frac{2}{3}\frac{q^2 a^2}{c^3}, \tag{8}$$

where the symbols have their usual meaning. Show that for a weak electron-ion interaction, as illustrated in the figure below,



the total energy emitted is given by,

$$W = \frac{\pi}{4}\frac{Z^2 e^6}{m_e^2 c^3 b^3}\frac{1}{v} \tag{9}$$

where the symbols have their usual meaning [9 points].

Useful constants

Boltzmann constant $k = 1.3806488 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}
Speed of light in a vacuum $c = 1/\sqrt{\mu_0 \epsilon_0} = 299\,792\,458$ m s^{-1}
Permittivity of free-space $\epsilon_0 = 8.85418782 \times 10^{-12}$ A^2 s^4 kg^{-1} m^{-3}
1 Mpc $\equiv 3.08568 \times 10^{24}$ cm

Useful identities

$e^{i\phi} = \cos\phi + i\sin\phi$
The similarity theorem: if $f(l)$ is the Fourier transform of $g(u)$, then $(1/|a|)f(1/a)$ is the Fourier transform of $g(au)$, where a is a constant and not 0.
$\int_0^{\pi/2} \cos^4\theta d\theta = 3\pi/16$
$d/dr[u(r)/v(r)] = [u'(r)v(r) - v'(r)u(r)]/v(r)^2$